# Non Intrusive Reduced Basis method applied to wind turbine simulation

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## Main goal

In an industrial context, parameter dependant PDEs may have to be solved numerically several times, and it can lead to very large computing times, for instance for:

- optimization paramaters fitting
- complex systems with uncertain scenarios

Non Intrusive Reduced Basis method (NIRB) is a quickly emerging field in applied mathematics and computational science. They enable high fidelity real-time simulations and **reduce the computational costs** thanks to the small Kolmogorov n-width [2]. These methods are decomposed in two stages: one **offline**, costly in time, and one **online**, much faster.

## Industrial context

**Code_Saturne** is a free computer simulation software in fluid mechanics developed by EDF, using Finite Volumes method. One of the applications of Code_Saturne aims to obtain an optimal energy production by wind-farm. To do so, a specific velocity positioned upstream of a wind turbine is computed using a Reynolds Averaged Navier-Stokes (RANS) Equation for a given input velocity ($uref$). To further reduce calculation times, **non intrusive reduced basis methods** are suitable such as the **two-grid method**. RANS method gives mean quantities in time, and comes from the averaged Navier-Stokes equations:

$$\operatorname{div} \overline{\mathbf{u}} = 0, \tag{1}$$

$$\frac{\overline{u}_i}{\partial t} + \sum_{j=1}^{3} \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = \overline{f}_i - \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \sum_{j=1}^{3} [\nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} - \frac{\partial \overline{u_i' u_j'}}{\partial x_j}], \tag{2}$$

deduced from the continuity and the momentum equations and the decomposition: $u_i = \overline{u}_i + u_i'$. Here, $\mathbf{u} = (u_1, u_2, u_3)$ is the velocity, $\rho$ is the fluid density, $p$ its pressure, $\nu$ the viscosity.

## Two-grid method

**The goal is to find a good approximation of the velocity on a fine mesh $\Omega_h$ using a velocity on a coarse mesh $\Omega_H$.**

To successfully apply reduced basis method, the Kolmogorov n-width must be very small. It means that the solution manifold $\mathcal{M}_h = \{\mathbf{u}_h(uref) \in X_h | uref \in \mathbb{R}^{Ntrain}\}$ for discrete solutions $\mathbf{u}_h(uref)$ of the RANS model on a fine mesh (of size $h$), may be approximated by a finite set of well-chosen solutions.

**High-fidelity model (RANS)**

**Get snaphots**

**1.** Run Code_Saturne as a **black-box** (no need to modify the code) to get discrete solutions for several parameters $uref$ well chosen.

Output: $X = (\mathbf{u}_h^1, \dots, \mathbf{u}_h^{Ntrain}) \in \mathbb{R}^{\mathcal{N} \times Ntrain}$

**Greedy algorithm,POD,PGD,...**

**Build a reduced basis by POD**

**2.** Set the correlation Matrix $C_{i,j} = \int_{\Omega_h} \mathbf{u}_h^i \cdot \mathbf{u}_h^j$,

**3.** Solve eigenvalues problem: $C\mathbf{\Psi}_h^i = \lambda_i \mathbf{\Psi}_h^i$, where $\mathbf{\Psi}_h^i \in \mathbb{R}^{Ntrain}$.

**4.** For $N$ ordered proper values ($\lambda_1 > \dots > \lambda_N$), calculate $N$ functions $\boldsymbol{\phi}_h^k = \sum_{j=1}^{Nrain} \Psi_h^{k,j} \mathbf{u}_h^k \; \forall k = 1, ..., N$,

**5.** Normalize: $\boldsymbol{\phi}_h^k = \frac{\boldsymbol{\phi}_h^k}{\sqrt{\lambda_k}}$.

Output: $(\boldsymbol{\phi}_h^k)_{k=1,...N}$

**NIRB**

**Reconstruct a fine approximation for a new parameter *uref***

**6.** A coarse mesh $\Omega_H$ is involved for the RANS resolution with $H >> h$ the size of the mesh. Consequently, the time cost to obtain $\mathbf{u}_H(uref)$ is low. From the following estimates [1]:

$$\left\| \mathbf{u}(\mathbf{x}, uref) - \sum_{k=1}^{N} (\mathbf{u}_H(uref), \boldsymbol{\phi}_h^k)_{L^2} \boldsymbol{\phi}_h^k \right\|_{H^1} \leq C_1 h + C_2 H^2 + C(N)\varepsilon \sim o(h) \text{ if } H^2 \sim h,$$

where $C_1, C_2$, and $C$ are constants independent of $h$ and $H$,

we deduce that $\mathbf{u}_H^N = \sum_{k=1}^{N} (\mathbf{u}_H(uref), \phi_h^k)_{L^2} \phi_h^k$ is a good approximation.

Output: $\mathbf{u}_H^N \in X_h^N = Span\{\mathbf{u}_h^1, \dots, \mathbf{u}_h^N\}$

## Application

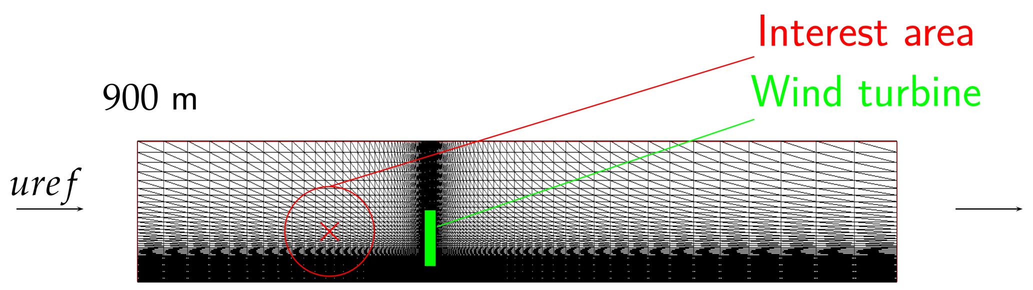


Figure 1: Mesh for one wind turbine

- ▶ 2D mesh with 6500 cells, thinner around the wind turbine.
- ▶ Characteristic length D: 126m, corresponds to the rotor diameter.
- ▶ Hub height: 95.6m.
- ▶ Wind turbine rotor is represented in the movement equation by adding a source term.
- ▶ Boundary Condition: $uref$ at the inlet.
- ▶ Initial Condition: $uref$ set in the domain.

## Results on one 2D wind turbine

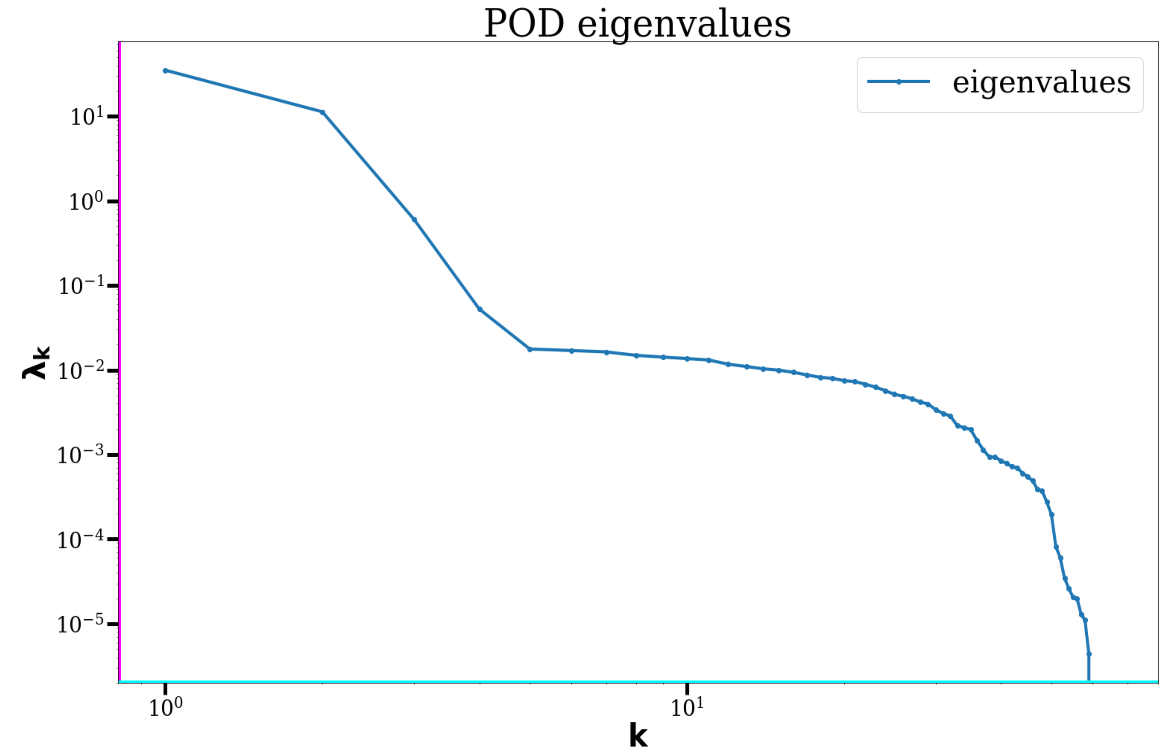


Figure 2: Decrease of the eigenvalues of the POD

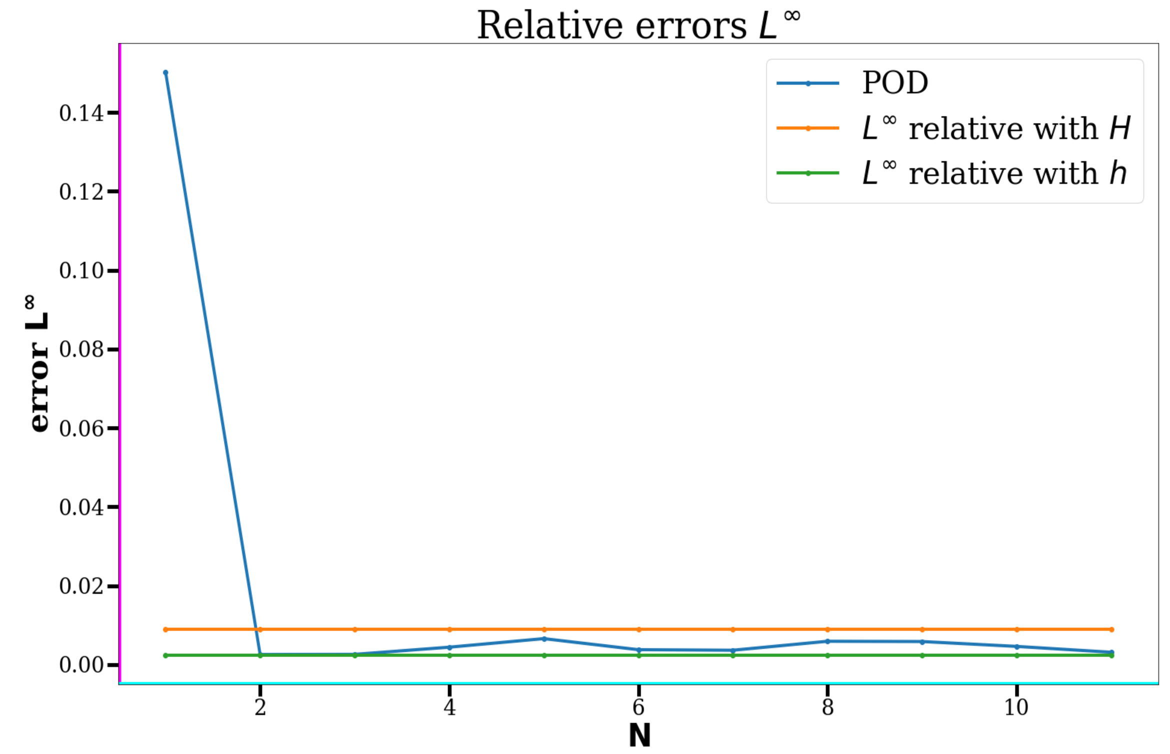


Figure 3: $L^\infty$ errors of the velocity on the interest area

- For $k = 3$, $I(k) = \frac{\sum_{j=1}^{k} \lambda_j}{\sum_{j=1}^{Ntrain} \lambda_j} \simeq 1$, which means that the eigenvalues of POD decrease enough to perform NIRB method.
- After $k = 5$, the decay is slowed down (Figure 2), so the error of the POD increases slightly (Figure 3).
- The relative error of $\left\| \mathbf{u}_{h/10} - \mathbf{u}_H^N \right\|_{L^\infty}$ is closer to the one given by $\left\| \mathbf{u}_{h/10} - \mathbf{u}_h \right\|_{L^\infty}$ rather than the one with $\mathbf{u}_H$ (Figure 3).

## Conclusions and perspectives

- We deduce from these results that the two-grid method may be applied to this case of application.
- An analysis on the size H of the coarse mesh should provide the adequate size to obtain the best approximation.
- **We plan to extend this method to the case of several wind-turbines.**

## References

[1] R. Chakir, Y. Maday, *A two-grid nite-element/reduced basis scheme for the approximation of the solution of parameter dependent P.D.E*, 2009. hal-00387405f.

[2] A. Buffa1, Y. Maday, A.T. Patera, C. Prudhomme and G. Turinici, *A priori convergence of the greedy algorithm for the parametrized reduced basis*, 2012. ESAIM: Mathematical Modelling and Numerical Analysis, EDP Sciences, 46 (3), pp.595-603.