Optimization of a Non-Intrusive Reduced Basis method (NIRB)

Mathematics of High-Performance Computing

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September 21, 2021

What is its purpose?
The two-grid method is non-intrusive

Industrial context $\rightarrow$ black box solver (BB)

Reduced basis methods useful for:

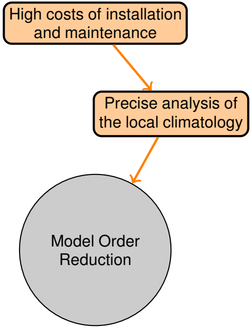
- Optimization parameters fitting
- High fidelity real-time simulations

Goal: Solve for several parameters a parameterized problem and reduce the computational costs

Offshore wind Farm

6	7	8
3	4	5
0	1	2

- Optimization
  of the wind turbines position
- High-fidelity RANS code
- Simulations costly in time



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⇑

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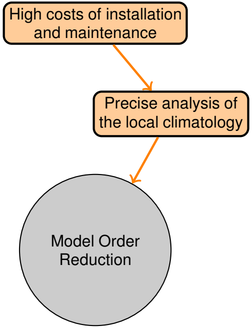

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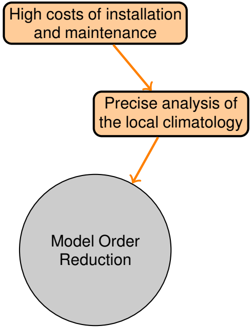

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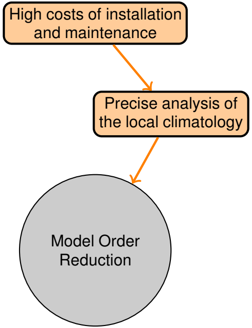

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NIRB approach: A two-grid method [1]

- Two meshes: $\mathcal{T}_h$ and $\mathcal{T}_H$.
- Two stages: One offline and one online.

- u_{hH}^N is the NIRB approximation
- $\mu \in \mathbb{R}^d$: varying parameter
- $u_H(\mu)$: Coarse solver solution ($H^2 \sim h$)
- $u_h(\mu_i)$: Snapshots on the fine mesh ($u_h(\mu_i) \in X_h^N$)

[1] R. Chakir, Y. Maday (2009). *A two-grid nite-element/reduced basis scheme for the approximation of the solution of parameter dependent PDE.*

Error estimates

Energy error estimate

$$\left\|u(\mu)-u_{hH}^N(\mu)\right\|_{H^1} \leq \overbrace{\varepsilon}^{T_1} + \underbrace{C_1 h}_{T_2} + \overbrace{C_2(N)H^2}^{T_3} \sim Ch \text{ if } H^2 \sim h$$

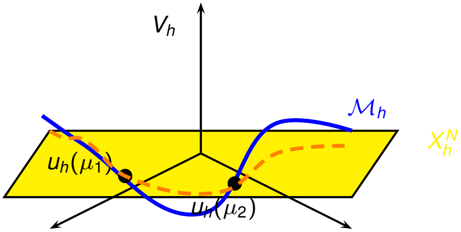
where C_1, C_2 are constants independent of h and H.[2]

[2] R. Chakir, Yvon Maday. *A two-grid finite-element/reduced basis scheme for the approximation of the solution of parameter dependent PDE.* 2009

[3] E. Grosjean, Yvon Maday. *An error estimate of the non-intrusive reduced basis method with finite volume schemes* 2021

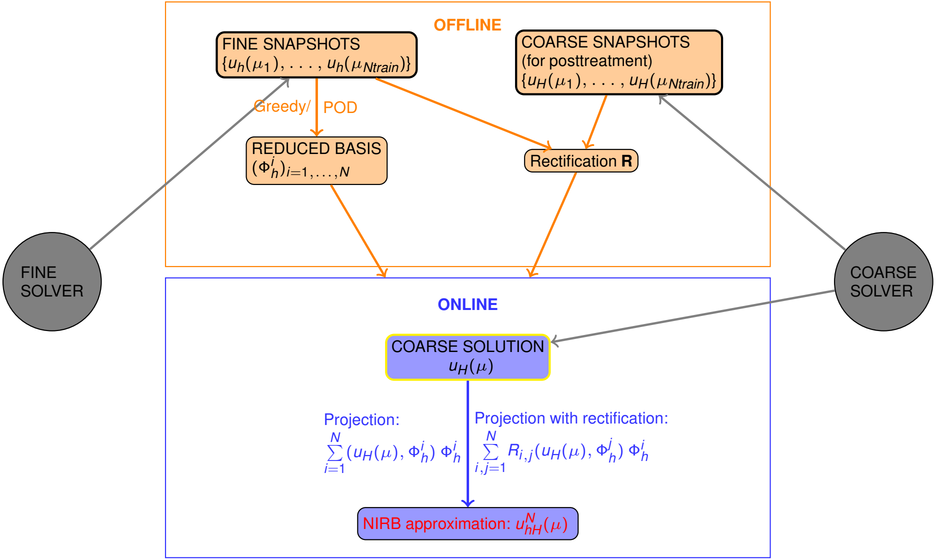
What is behind this method?

The Kolmogorov n-width must be small [4]
$\mathcal{M}_h = \{u_h(\mu) \in V_h | \ \mu \in \mathcal{P}\}$ is a subset of a Banach space V_h.



[4] A. Buffa, Y. Maday, A.T. Patera, C. Prud'homme, and G. Turinici, *A Priori convergence of the greedy algorithm for the parameterized reduced basis.* 2010

NIRB – OFFLINE/ONLINE



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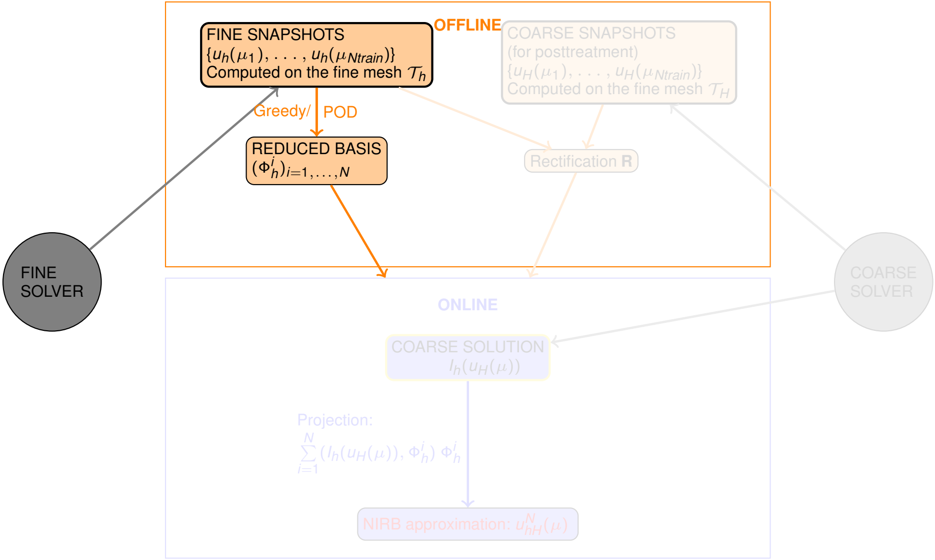


NIRB – OFFLINE/ONLINE

OFFLINE

FINE SNAPSHOTS
$\{u_h(\mu_1), \ldots, u_h(\mu_{Ntrain})\}$
Computed on the fine mesh $\mathcal{T}_h$

COARSE SNAPSHOTS
(for posttreatment)
$\{u_H(\mu_1), \ldots, u_H(\mu_{Ntrain})\}$
Computed on the fine mesh $\mathcal{T}_H$

_ POD $\longrightarrow L^2$ orthonormalization.

_ Greedy algorithm with a Gram-Schmidt procedure

+ Eigenvalue problem: $\forall v \in X_h^N, \int_\Omega \nabla \Phi_h \cdot \nabla v = \lambda \int_\Omega \Phi_h \cdot v,$

$\longrightarrow L^2(\Omega)$ and $H^1(\Omega)$ orthogonalization.

$\mathbf{X_h^N} = \mathbf{Span}\{\Phi_h(\boldsymbol{\mu}_1), \ldots, \Phi_h(\boldsymbol{\mu}_N)\}$

...ctification **R**

FINE SOLVER

COARSE SOLVER

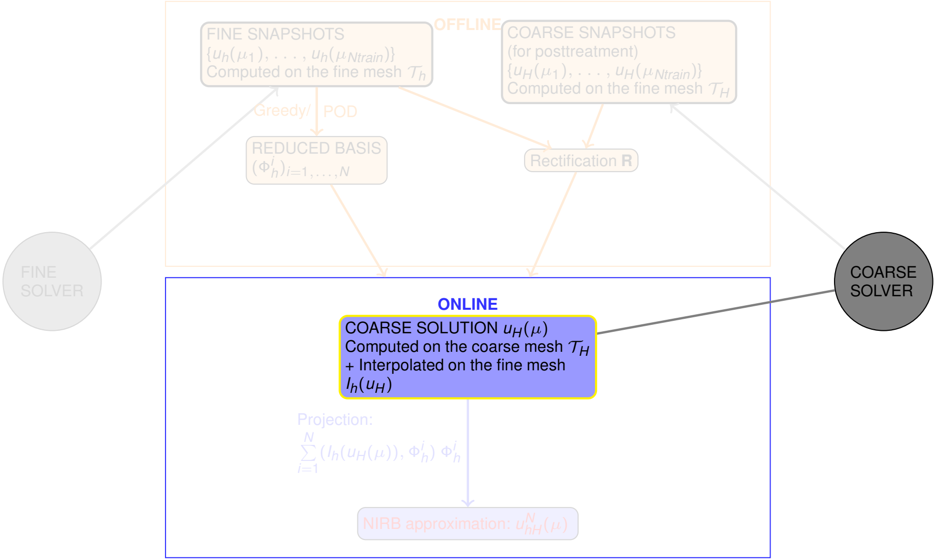
ONLINE

COARSE SOLUTION
$I_h(u_H(\mu))$

Projection:
$\sum_{i=1}^{N} (I_h(u_H(\mu)), \Phi_h^i)\ \Phi_h^i$

NIRB approximation: $u_{hH}^N(\mu)$

NIRB – OFFLINE/ONLINE



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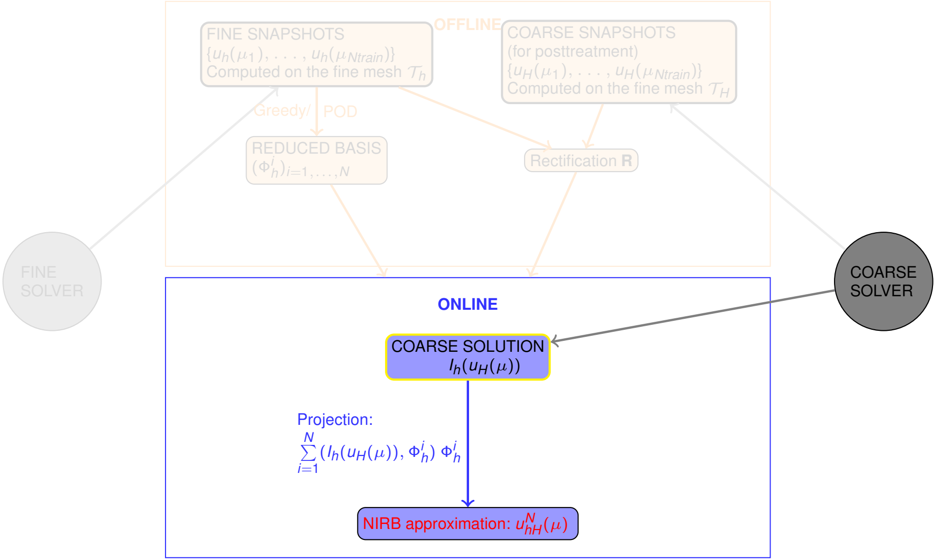

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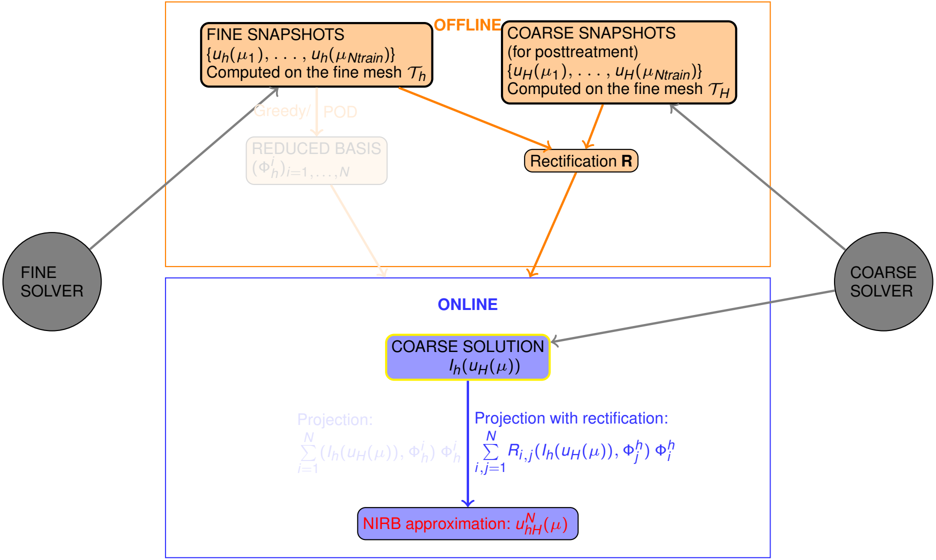


1. COARSE SNAPSHOTS
   $\beta_i^j = (I_h(u_H(\mu_i)), \Phi_h^j)$.
   Fine coefficients: $\alpha_i^j = (u_h(\mu_i), \Phi_h^j)$.

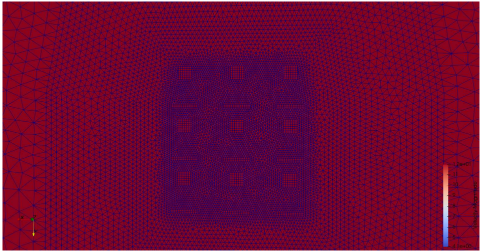
2. $R^i = (\beta^T \beta + \lambda \times I_N)^{-1} \beta_i \alpha_i$,

3. $\widetilde{\alpha_i(\mu)} = \sum_j R(i,j)(I_h(u_H(\mu)), \Phi_h^j)$.

[5] R. Chakir, Y. Maday, P. Parnaudeau (2019). *A non-intrusive reduced basis approach for parametrized heat transfer problems.*

The application

Full domain:
(Ox,Oy) $\rightarrow [-4400, 4400]^2$
(Oz) $\rightarrow$ 0 to 1000
D=150
Rotor: z=100m
μ: Velocity at the entrance
RANS
3.5 M nodes



Visualization

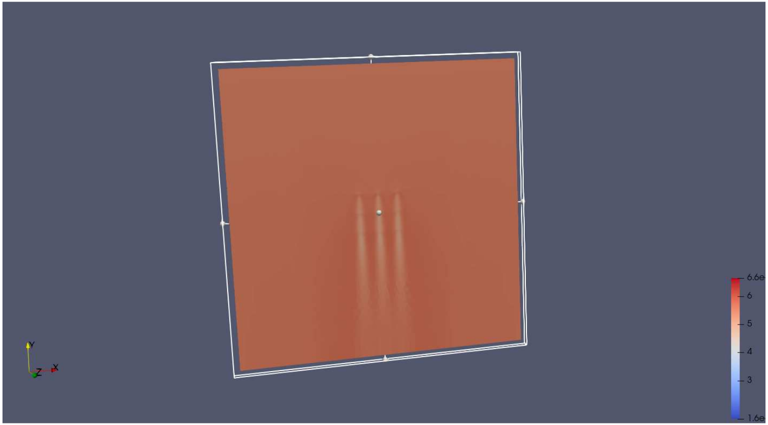


Figure: Wind Turbines

The parameters

Meshes size:

- One fine (3 637 438 nodes)
- One coarse (686 504 nodes)

Zoom around the wind turbines (Oz): 25 to 200

Wind angle and magnitude

- Angle from 0 to 45
- Magnitude from 3 to 24

Number of snapshots = 220 ($\times$ 9=1980)

Magnitude 3, angle 0

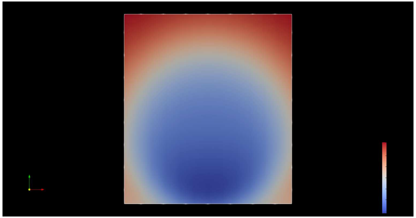


Figure: windturbine 0

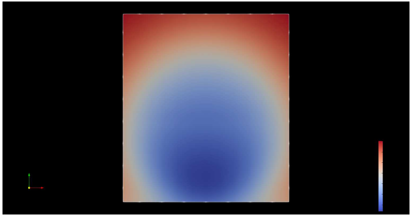


Figure: windturbine 8

Recall windturbines

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H^1 relative errors

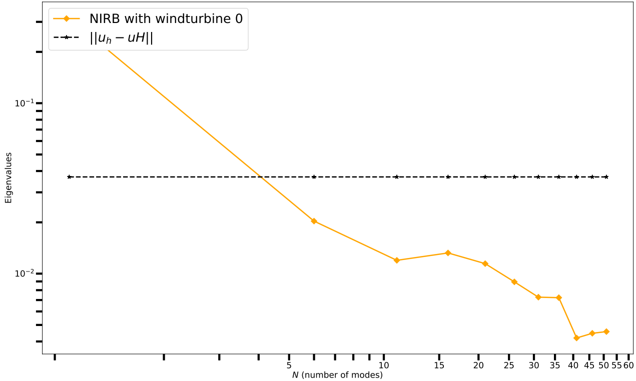

Figure: Wind Turbine 0 (magnitude= 23, degre= 5)

H^1 relative errors

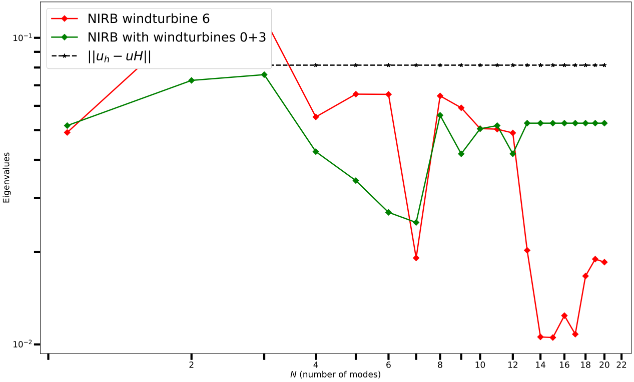

Figure: Wind Turbine 6 (magnitude=5, degre=0)

Results

H^1 relative errors

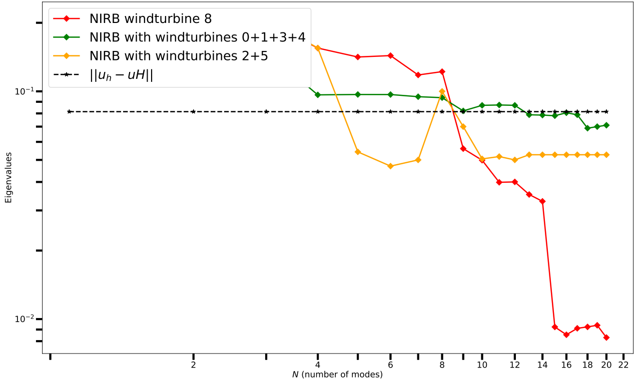

Figure: Wind Turbine 8 (magnitude=23, degre=0)

Ongoing projects

- A posteriori estimate for the rectification matrix
- Deterministic learning process with a smaller condition number

Conclusions

- Fine approximation from a coarse solver solution
- Optimal approximation for elliptic equations
- Several applications
- Several posttreatments (methods to stabilize the condition number)

Thank you for your attention!

Questions?

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Bibliography

1. R. Chakir, Y. Maday (2009). *A two-grid nite-element/reduced basis scheme for the approximation of the solution of parameter dependent PDE.*
2. R. Chakir, Yvon Maday. *A two-grid finite-element/reduced basis scheme for the approximation of the solution of parameter dependent PDE.* 2009
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5. R. Chakir, Y. Maday, P. Parnaudeau (2019). *A non-intrusive reduced basis approach for parametrized heat transfer problems.*