Continuous optimization
ENT 305

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And to sum up the courses ...

	Necessary conditions	Sufficient conditions
Abstract formulation (exist.)		if K compact, $f \in C^0(K)$ then at least one solution
		if K closed, $f \in C^0(K)$, coercive then at least one solution

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	Necessary conditions	Sufficient conditions
No constraints $K = \mathbb{R}^d$ (opt.)	if $\overline{x}$ local sol., $f \in C^2(K)$ then, $D^2 f(\overline{x})$ is positive semi-def.	if $f \in C^2(K)$, $\nabla f(\overline{x}) = 0$, $D^2 f(\overline{x})$ positive def. then $\overline{x}$ local sol.
Affine constraints	$\overline{x}$ local sol. then KKT	f convex, then KKT=global sol.
Non-linear constraints	$\overline{x}$ local sol., LICQ then KKT	f convex, h affine, g convex, then KKT=global sol.

Exercice

Consider the following optimization problem:

$$\inf_{(x,y,z)\in\mathbb{R}^3} \exp(x+y+z-2)-5x-2y+\gamma z, \quad \text{subject to: } \begin{cases} x^2+\frac{1}{2}y+z \leq \frac{3}{2} \\ x \geq 0 \\ y \geq 0 \\ z \geq 0. \end{cases}$$

For the moment $\gamma = 0$. We consider the point $(\bar{x}, \bar{y}, \bar{z}) = (1, 1, 0)$.

1. Prove that $(\bar{x}, \bar{y}, \bar{z})$ is feasible and satisfies the KKT conditions. Deduce that it is a global solution to the problem.
2. We allow now for an arbitrary value of γ. Find the set of values of γ for which $(\bar{x}, \bar{y}, \bar{z})$ still satisfies the KKT conditions.

Exercice

We consider the following optimization problem:

$$\inf_{(x,y)\in\mathbb{R}^2} \exp\left(2x-2y^2\right)-2x+3y, \quad \text{subject to:} \begin{cases} x+2y = 3 \\ x+y \geq 2 \\ 2x+5y \leq 10. \end{cases}$$

Are the KKT conditions verified at the point $(1,1)$?

Exercice

- Formulate the optimization problem for a very simple model of a microgrid.

- **Microgrid:** a set consisting of the following elements:
  - a small-size **electrical load** (a building, a few houses)
  - a source of **renewable energy** (solar panels, a wind turbine)
  - a **storage** device (Energy Storage System, a battery)
  - a **macrogrid** (an access to a large-scale energy network).

Exercice

- **Time scale:** the decisions are taken **every hour** during the **day**.
- **Optimization variables** (at each time step) :
  - the amount of energy to be **stored** or **withdrawn** from the battery
  - the amount of energy **bought** or **sold** to the network.
- **Contraints:**
  - nonnegativity of variables
  - evolution of the battery
  - storage capacity of the battery.
- **Cost function:**
  - Cost of the energy bought on the network...
  - minus the cost of the energy sold on the network.

Deterministic model

- Horizon: 24 hours, stepsize: 1 hour.
  Optimization over $T = 24$ intervals.

- Optimisation variable :
  - $a(s)$: amount of electricity bought on the network ($s = 1, ..., T$).
  - $v(s)$: amount of energy sold on the network ($s = 1, ..., T$).

- Parameters:
  - $x(s)$: state of charge of the battery at time s, $s = 1, ..., T + 1$
  - $d(s)$: net demand of energy (load minus solar production) at time s, $s = 1, ..., T$.
  - $P_a(s)$: unitary buying price of energy at time s
  - $P_v(s)$: unitary selling price of energy at time s
  - x_{max}: storage capacity of the battery.

Deterministic model

- Contraints:
  - $x(s+1) = x(s) - d(s) + a(s) - v(s), \forall s = 1, ..., T$
  - $x(1) = 0$
  - $a(s) \geq 0, \forall s = 1, ..., T$
  - $v(s) \geq 0, \forall s = 1, ..., T$
  - $0 \leq x(s) \leq x_{max}, \forall s = 1, \dots T + 1.$
- Cost function to be minimized:

$$J(x, a, v) = \sum_{s=1}^{T} \Big(P_a(s)a(s) - P_v(s)v(s) \Big).$$