Optimization Project in Energy
ENT306

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Indices

- Set of dams $\mathcal{I}$
- Set of rivers $\mathcal{E} \subset \mathcal{I} \times \mathcal{I}$:
  $$(i, j) \in \mathcal{E} \iff \text{river flows from dam } i \text{ to dam } j.$$
- Set of time intervals: $\{1, ..., T\}$.

Optimization variables

- $q_{i,t}$: water level of dam i at the beginning of time interval $t \in \{1, ..., T + 1\}$
- $x_{i,t}$: amount of water exploited at dam i during time interval $t \in \{1, ..., T\}$
- $y_{(i,j),t}$: amount of water transported over the river (i, j) during the time interval $t \in \{1, ..., T\}$
- $z_{i,t}$: amount of water exploited at dam i during the time interval $t \in \{1, ..., T\}$, not transported to any other dam.

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Parameters

- $P_{i,t}$: precipitation at i, during the time interval t
- Q_i: storage capacity of dam i
- K_i: initial level of dam i
- D_t: electricity demand during the time interval t

Functions

- $f_i : x \mapsto f_i(x)$: exploitation cost on a given time interval at dam i, as a function of the amount of exploited water x.
- $g_i : x \mapsto g_i(x)$: electricity production as a function of the amount of exploited water at dam i.

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Cost function

$$\min_{q,x,y,z} \sum_{t=1}^{T} \sum_{i \in I} f_i(x_{i,t}).$$

Constraints

- Nonnegativity of the variables:

$$q_{i,t} \geq 0, \quad x_{i,t} \geq 0, \quad y_{(i,j),t} \geq 0, \quad z_{i,t} \geq 0.$$

- Bounds:

$$q_{i,t} \leq Q_i, \quad \forall t \in \{1, ..., T+1\}.$$

- Initial condition:

$$q_{1,i} = K_i.$$

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- Demand satisfaction:

$$\sum_{i \in \mathcal{I}} g_i(x_{i,t}) = D_t, \quad \forall t \in \{1, ..., T\}.$$

- Evolution of the water level in each dam:

$$q_{i,t+1} = q_{i,t} + P_{i,t} - x_{i,t} + \sum_{\substack{j \in \mathcal{I} \\ (j,i) \in \mathcal{E}}} y_{(j,i),t}, \quad \forall t \in \{1, ..., T\}.$$

- Amount of exploited water:

$$x_{i,t} = \sum_{\substack{j \in \mathcal{I} \\ (i,j) \in \mathcal{I}}} y_{(i,j),t} + z_{i,t}, \quad \forall t \in \{1, ..., T\}.$$

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Dynamic programming.

We parametrize the problem by

- the initial time interval t
- the initial level of water in every dam $q \in \mathbb{R}^{\mathcal{I}}$.

Let $V(t, q)$ denote the corresponding optimal cost.

We have $V(T+1, q) = 0$.

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Dynamic programming principle

Let $t \in \{1, \ldots, T\}$ and let $q \in \prod_{i \in \mathcal{I}} [0, Q_i]$. Then

$$V(t, q) = \inf_{\substack{q' \in \mathbb{R}^{\mathcal{I}}, x \in \mathbb{R}^{\mathcal{I}}, \\ y \in \mathbb{R}^{\mathcal{E}}, z \in \mathbb{R}^{\mathcal{I}}}} \left(\sum_{i \in \mathcal{I}} f_i(x_i) \right) + V(t+1, q'), \qquad (DP(t,q))$$

subject to:

- Non-negativity: $q'_i \geq 0$, $x_i \geq 0$, $y_{(i,j)} \geq 0$, $z_i \geq 0$.
- Bounds: $q'_i \leq Q_i$.
- Demand: $\sum_{i \in \mathcal{I}} g_i(x_i) = D_t$.
- Conservation: $q'_i = q_i + P_{i,t} - x_i + \sum_{\substack{j \in \mathcal{I} \\ (j,i) \in \mathcal{E}}} y_{(j,i)}$.
- Exploitation : $x_i = \sum_{\substack{j \in \mathcal{I} \\ (i,j) \in \mathcal{I}}} y_{(i,j)} + z_i$.

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Remarks.

- Why does it work?
  → The level of water in the dams at time t is a **sufficient information** to take optimal decisions from t until the end of the optimization process.
  → Knowing the level of water in the dams at time t, one can completely **forget** what happened in the past.

- The dynamic programming principle characterizes globally optimal solutions, even if the original problem is non convex.

- In the practical implementation of the method, one needs to **discretize** the variable q. The number of discretizetion points grows **exponentially** with the number of dams. This phenomenon is called **curse of dimensionality**.